Megaliths and Mathematics

by ALEXANDER THOM

We are hearing more and more about the mathematical and astronomical aspects of megalithic monuments, a subject that has often been ventilated in the pages of this journal. Now we have a book by G. S. Hawkins and J. B. White entitled Stonehenge Decoded *which is described as 'the account of how computer analysis revealed the stones as a sophisticated astronomical observatory'; this will be reviewed by Professor R. J. C. Atkinson in our next issue. Dr Alexander Thom was Professor of Engineering Science in Oxford from* 1945 *to* 1961, *and now, living in retirement in Ayrshire, is an Emeritus Fellow of Brasenose. All his life he has been interested in the measurements used by the megalith builders and has written many articles on this subject. Here he summarizes for readers of* ANTIQUITY *his general views on this whole vexed matter.*

IT is becoming apparent that megalithic man possessed and used a considerable knowledge of geometry. As more of his constructions are unravelled, we obtain an increasing appreciation of his attainments. Undoubtedly he also observed the heavenly bodies and used them to tell the time of day or night and to tell the day of the year. To take geometry first, let us look at the various shapes which, in his hands, a ring of stone could take. To understand these rings fully it is necessary to appreciate that he used extensively a very precise unit of length—the megalithic yard (MY). The exact length of this unit has become known to us by an examination of simple circles and flattened circles. When the author produced the first batch of circle diameters there was no universally accepted statistical analysis for the determination of the reliability of a quantum such as the suggested value for the megalithic yard. Then Broadbent produced two papers providing exactly the methods required to find, from a set of measurements, the most probable value of the quantum and the probability level at which it could be accepted [1,2]. This last is very important because Hammersley had shown that almost any random set of (say) diameters will yield an apparent unit of some sort.

Logically a sound approach would be to use the measurements to test an *a priori* value of the quantum, but in the case of the megalithic yard we have no *a priori* value. The unit must come from the data themselves and Broadbent's second paper provides for this case. It is sufficient to say that the accumulated data, much of which I set out in 1962, stands up to Broadbent's analysis, thus establishing definitely that the unit exists and that its value is just over 2·72 ft. [3]. The result is the same whether the unit is derived from the English or the Scottish circles. The analysis gives us two interesting by-products: (a) the precision of the measurements does not decrease with length, and (b) the builders of the circles measured to the centres of the stones in a ring. Exceptions to this rule occur in those cases where the ring was of closely spaced stones forming a retaining-wall holding rubble filling. Then it would only be natural to measure to the more or less regular side of the wall outside the tumulus or to the inside of a wall forming a cell inside. It has also been shown that half and perhaps quarter yards were often used in alignments [3]. Later work supports this subdivision of the yard, but no trace of a subdivision into three is apparent.

In their desire to use integral multiples of the yard the incommensurability of π posed a problem. One soon notices how many of the smaller circles have a diameter of about 22 ft.

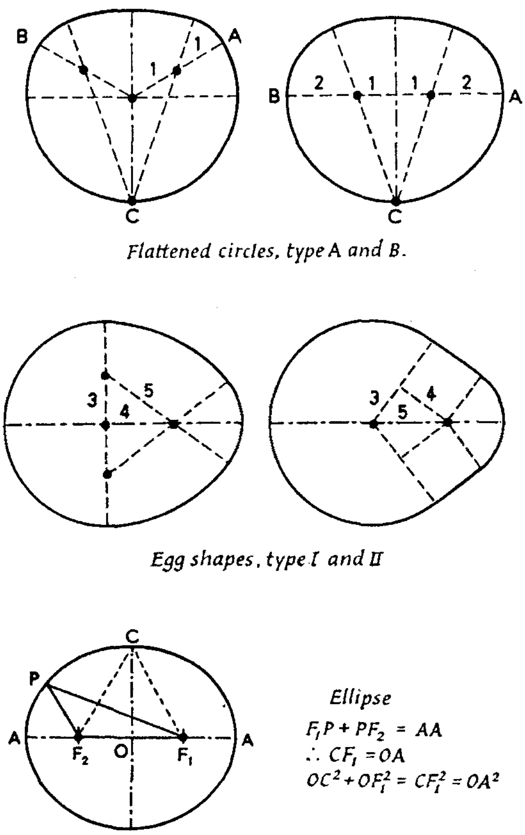


Fig. 1. *Classes of megalithic rings*

These are 8 megalithic yards in diameter and so with $\pi = 3\frac{1}{8}$ the circumference would be 25. The importance of 25 lies in megalithic man's use of a larger unit of 10, subdivided again into halves and quarters. It seems likely that for longer distances he used measuring rods $2\frac{1}{2}$ yd. (6·80 ft.) long.

Some of the types of rings used are shown (FIG. 1). There are at least 25 examples of 'flattened circles' Types A and B still in existence, many of which have been surveyed [4, 5]. There are 9 sites known with egg-shaped rings Type I and II. Type I is the commonest but both are based on Pythagorean triangles or triangles which are nearly Pythagorean. The favourite is the 3,4,5 triangle which, from earliest times, has been used to set out a right-angle. Having laid out two of these triangles back to back we have established 4 points on the ground (FIG. 1). The egg shape can now be constructed by scribing four arcs centred on these four points. Type II differs in the placing of the triangles and in having only two arcs joined by straight lines parallel to the side of the triangle. It will be evident in both types that once the first arc is drawn the others follow, their radii being determined by that of the first. Further, if the radius chosen for the first arc drawn is an integral number of yards then, since the sides of the triangle are integers, the other radii will also be integers. An example is given which shows the inner ring at 'The Druid Temple' near Inverness (FIG. 2).

When the triangles have been drawn any desired value can be chosen for the radius of the first arc, so a large variety of egg shapes can be drawn. It appears that the value actually chosen was such as to make, in nearly all cases, the perimeter of the ring as closely as possible a multiple of $2\frac{1}{2}$ yd.

The most important of the egg-ring sites is Woodhenge (FIG. 3). Here the triangle was 12, 35, 37, an exact Pythagorean triangle, set out in units of the half yard. By a little trigonometry we can show that with this triangle the relation between the radius chosen for the large end and the perimeter P is:

$$r = (P - 9{\cdot}08)/2\pi$$

Using this, the values of r were found corresponding to P = 40, 60, 80, 140 and 160 yd. To check that the above was the construction actually used at Woodhenge a careful large-scale survey was made and tested with a steel tape. The geometrical design, with the values of r as found above, was drawn carefully on tracing-paper and superimposed. The result is shown to a small scale in the figure. Further details regarding Woodhenge geometry and other egg-shaped rings have been set out elsewhere [6].

These triangles were also used for constructing ellipses. The easiest way to draw an ellipse on the ground is to drive two stakes at the points F_1 and F_2 chosen for the foci. A rope having a length equal to the required major axis has its ends tied to the stakes. The outline can then be drawn by a third stake slid round the rope. When the third stake is at C (FIG. 1) we see that the semi-axes major and minor and half

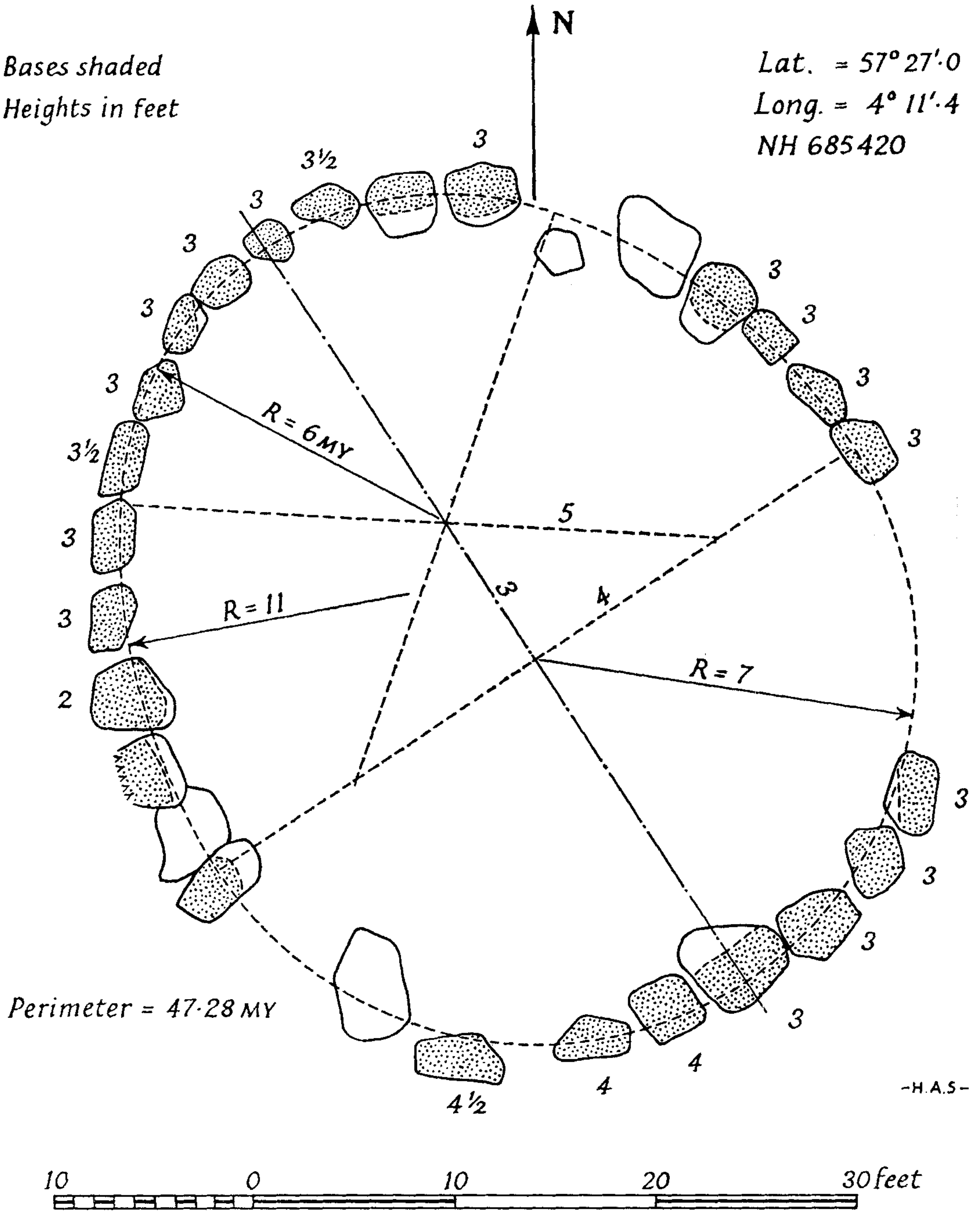


Fig. 2. Druid Temple (Inner Ring) near Inverness

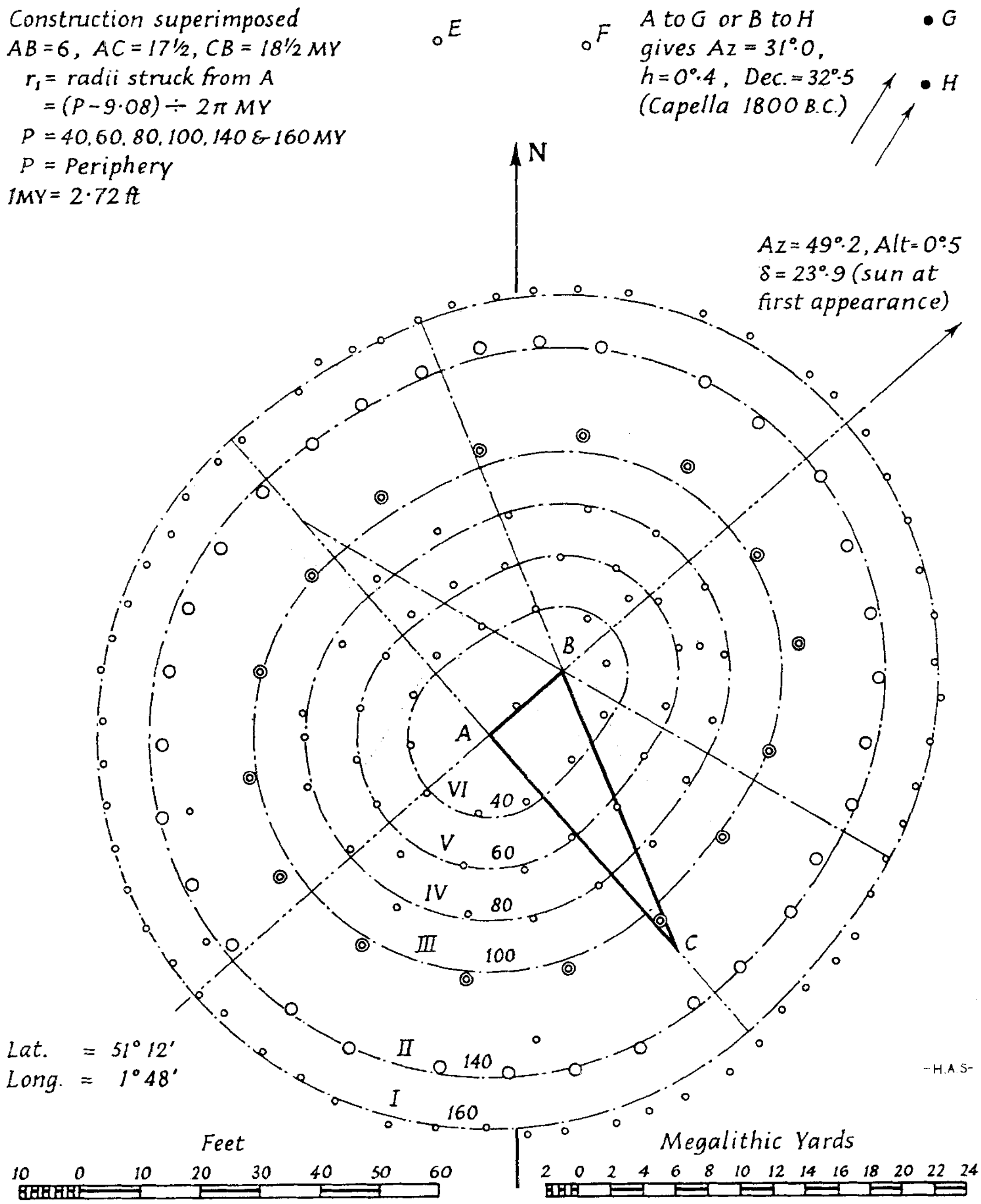


Fig. 3. Woodhenge

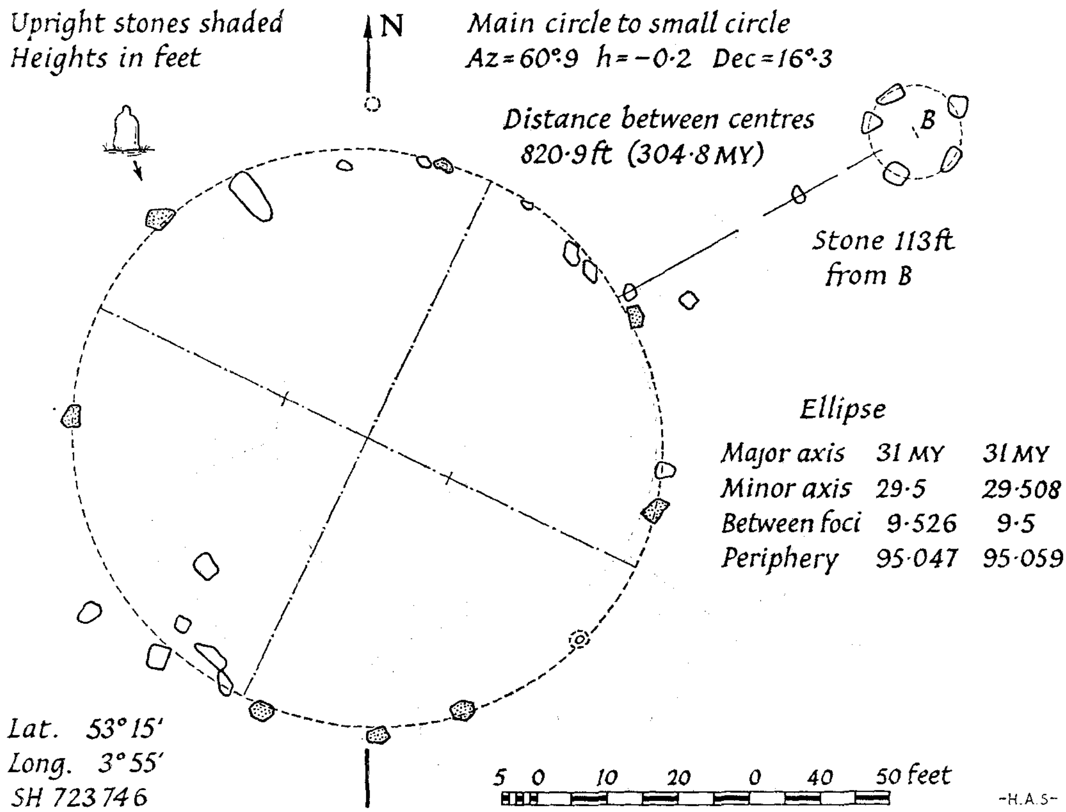


Fig. 4. *Penmaenmawr*

the focal distance OF_1 form a right-angled triangle. So if all the dimensions are to be integers we again require a Pythagorean triangle.

As an example take the 'circle' above Penmaenmawr (FIG. 4). Here the major axis is 31 MY and the distance between the foci is 9½ MY. This makes the minor axis equal to $\sqrt{(870{\cdot}75)}$ or 29·508, which on the ground is indistinguishable from 29½. It can be shown by calculation that the perimeter of the ellipse so drawn is 95·06 which is remarkably near 95. By a study of this circle we have, in fact, found another triangle which is almost Pythagorean since $19^2 + 59^2$ is 3842 and 62^2 is 3844. But how did they get at the same time dimensions which made the perimeter 95?

Of the nine ellipses surveyed up to the present by the author, only one fails to have its perimeter close to a multiple of 2½ MY. Today we would use a digital computer to discover ellipses with these properties. To do it by trial and error must have been a prodigious task.

A very interesting ring of compound type occurs in Wales at Moel Ty Ucha (FIG. 5). The details of the geometry are given (FIG. 6). With the radii of the two construction circles 4 and 7 MY the radius of the long arcs will be found to be 13½ MY. Trigonometry shows that the exact length is 13·503 and that the perimeter is 42·85. It is remarkable that with this beautiful construction these people succeeded in finding dimensions which made all radii integral and made the perimeter so close to 42½. The construction is superimposed on an accurate survey (FIG. 5). The agreement is much too

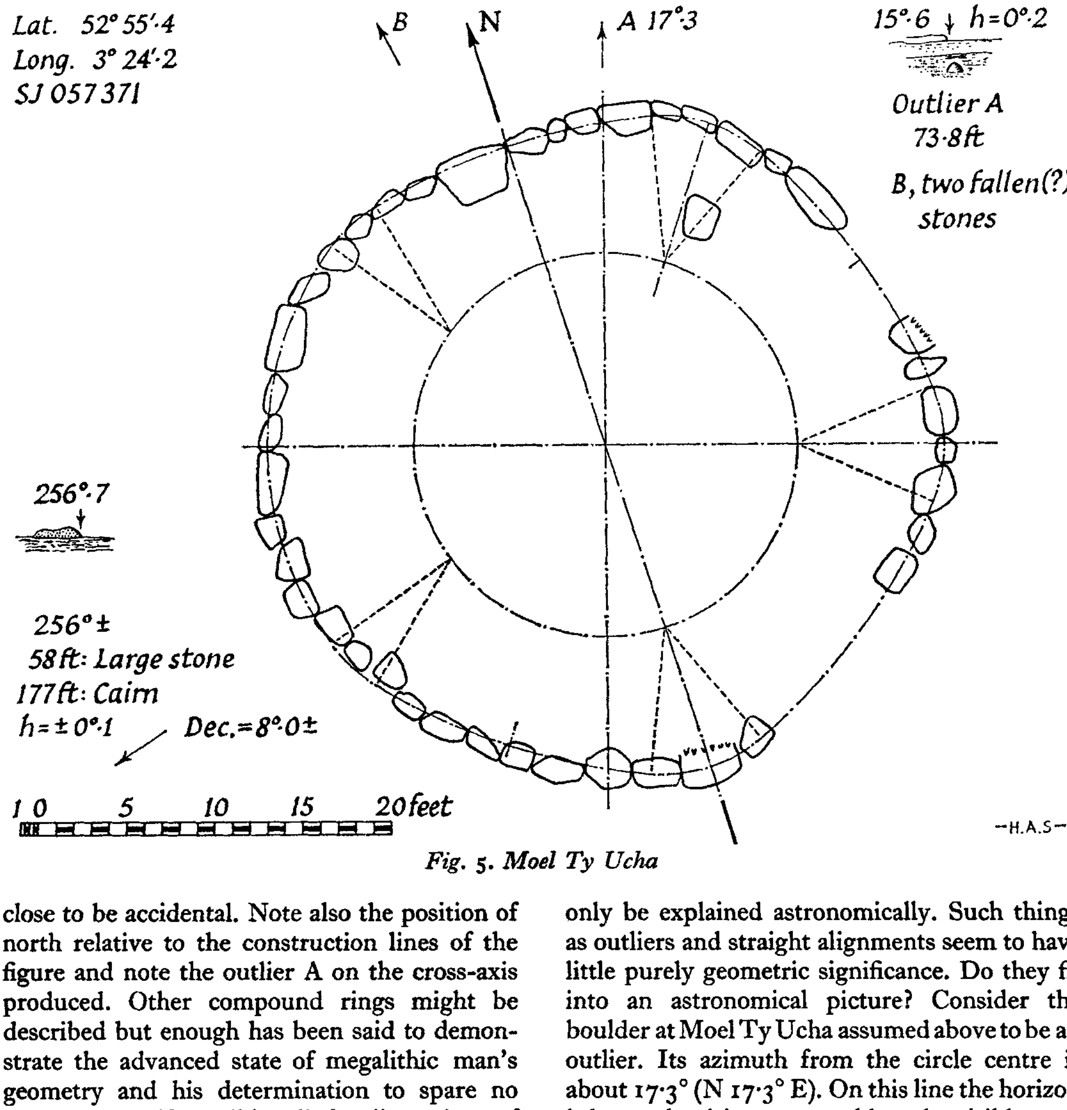


Fig. 5. Moel Ty Ucha

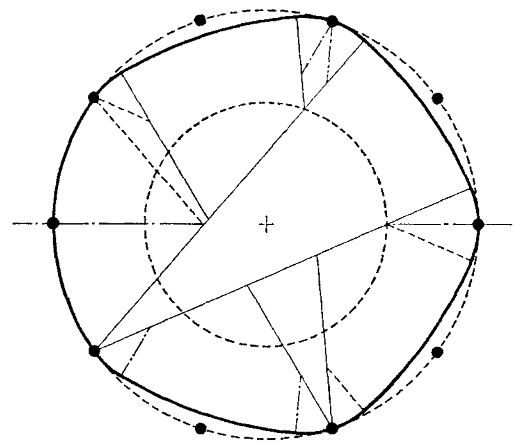
close to be accidental. Note also the position of north relative to the construction lines of the figure and note the outlier A on the cross-axis produced. Other compound rings might be described but enough has been said to demonstrate the advanced state of megalithic man's geometry and his determination to spare no pains, to get, if possible, all the dimensions of his figures multiples of the yard or half yard. One can only surmise that, having no pen and paper, he was building in stone a record of his achievements in geometry and perhaps also in arithmetic.

ASTRONOMY

Consider now the evidence that these people observed and used astronomical phenomena. A number of the most impressive sites such as Callanish, Temple Wood and Duncracaig can only be explained astronomically. Such things as outliers and straight alignments seem to have little purely geometric significance. Do they fit into an astronomical picture? Consider the boulder at Moel Ty Ucha assumed above to be an outlier. Its azimuth from the circle centre is about 17·3° (N 17·3° E). On this line the horizon is low and a rising star would not be visible until it had attained what is called its 'extinction angle' [9]. In megalithic times the first magnitude star Deneb would, in this sense, 'rise' when its altitude was about 1·4° and it would then be exactly on the line indicated. This isolated example is perhaps by itself unimpressive and might be accidental and so as many sites as possible have to be examined and the data subjected to strict statistical tests to find the probability that outliers and alignments were associated with first magnitude stars. This was attempted by the author in 1955 [4]. Using severe terms of reference for selecting the material the result was definite—there was a strong probability that stars were used as well as the sun. Unfortunately the importance of the extinction angle was not realized with the result that too early a mean date was found for the country as a whole. A repeat calculation taking into account the extinction angle and many new data shows a later date, but is not yet complete. It has shown that the relation between extinction angle and magnitude derived from megalithic sites agrees with that given by Neugebauer [9].

In megalithic times there were two methods available of telling the time: by the rising and setting of certain stars uniquely indicated by outliers, and by the transit of the sun or stars over the meridian. For the second method great slabs, or sometimes rows of slabs, were erected truly north and south. One can today watch the sun's shadow on these stones and so determine local apparent noon to within a few minutes. There are over a dozen such sites still capable of being used.

Because of the remarkable accuracy with which the site at Ballochroy can be operated one thinks of it as easily the best megalithic solstitial observatory [7]. This is only one of the large number of sites with indications for the solstitial sun, but other dates in the calendar were also important. It seems that the year was divided into 8 equal parts. In this connexion the most impressive evidence is the number of indicators for a declination between $+0{\cdot}4^\circ$ and $+0{\cdot}8^\circ$. If these are equinoctial indicators why do we not find zero declination? The reason given below is completely convincing. If we calculate the two dates in the year when, in megalithic times, the sun had a declination of $+0{\cdot}5^\circ$ we find that these are separated by exactly half a year. Thus megalithic man's 'equinox' occurred when the sun's declination was $+0{\cdot}5^\circ$. These dates, with the solstices, divide the year into 4. To subdivide again we need for example indicators for May Day, 46 days, i.e. one eighth of a year, before the summer solstice, and for Candlemas, 46 days after mid-winter. We find many lines giving



Construction: Set out outer circle with radius 7MY. Divide this circle into ten equal arcs. Set out inner circle with 4MY and draw the five 'corner-arcs' with crs. on the inner circle. Draw the four flat arcs with crs. on the large circle. The radii of these arcs will be found to be 13½ MY.

Fig. 6. Moel Ty Ucha—the geometrical construction

the required declinations. This matter is to some extent discussed [7], but since this was written much more confirmatory material has turned up, together with a group of lines for delineations $\pm 21\tfrac{1}{2}^\circ$. These are not explained satisfactorily by star positions but they are approximately the sun's declination 23 days before and after the equinoxes. This is not so well established as the May Day, Candlemas dates but support comes from the 5 or 6 lines showing a date 23 days before or after the equinoxes. These taken together indicate that the year was divided into 16 parts. Perhaps it should be pointed out that the accuracy with which a date may be determined from the position of the setting sun is in Scotland nearly twice as great as that in the tropics. But the solstices are the most difficult dates to determine in this way. The sun has then its maximum declination north or south and so its setting position is changing so slowly that only with a site like Ballochroy is there any hope of picking the exact day. It can however be shown that the sophisticated calendar which was in use, by linking the solstice with the more easily determined equinox, would determine midsummer day exactly. The calendars in the outlying districts could have been synchronized by signal fires such as were still being lit this century on midsummer night on the hilltops in various parts of Europe.

In the sense that *solstice* means a standing still of the sun's declination the moon might be said to have four solstices. The plane of the lunar orbit is inclined at some 5° to the ecliptic which is the path of the sun amongst the stars. The line of intersection of the two planes (the line of nodes) rotates slowly completing a revolution in about 18·6 years. This means that we sometimes see the moon 5° north of the sun's path and sometimes 5° south. It follows that the rising point of say the full moon nearest to midwinter oscillates back and forward along the horizon by some 20°. The extremes of this oscillation, the lunar solstices, are marked at some 30 places in Britain. Some of these sites are arranged like Ballochroy and give the maximum lunar declination with such great accuracy that we can be perfectly certain that they were set up as lunar observatories. They give an entirely independent mean date of 1800 ± 100 B.C.

Hawkins [8] points out that a 56-year cycle is better for eclipse prediction than an 18-year cycle, 56 being close to 3 × 18·6. Thus the 56 Aubrey holes at Stonehenge could have been used for markers to keep track of the position of the current year in the eclipse cycle. Since there was free communication among the communities why did each have to do its own eclipse prediction? Was this work eventually centred on Stonehenge?

We need more information. We need many more accurate surveys particular attention being paid to hill horizons. With our present lack of knowledge of cup and ring marks we cannot exclude these from the study. The existing sketches must be replaced by accurate plans accurately orientated. It is useless to make inferior surveys of the work of a people whose linear metrology was of a very high order.

NOTES

[1] S. R. Broadbent, 'Quantum Hypothesis', *Biometrika*, 42, 1955, 45.

[2] Broadbent, 'Examination of a Quantum Hypothesis Based on a Single Set of Data', *Biometrika*, 43, 1956, 32.

[3] A. Thom, 'The Megalithic Unit of Length', *J. R. Statist. Soc. A.*, 125, pt. 2, 1962, 243.

[4] Thom, 'A Statistical Examination of the Megalithic Sites in Britain', *J. R. Statist. Soc. A.*, 118, 1955, 275.

[5] Thom, 'The Geometry of Megalithic Man', *Math. Gazette*, 45, 1961, 83.

[6] Thom, 'The Egg-Shaped Standing Stone Rings of Britain', *Arch. Int. d'Hist. des Sciences*, No. 56–7, 1961.

[7] Thom, 'The Solar Observatories of Megalithic Man', *J. Brit. Ast. Soc.*, 64, No. 8, 1954, 396; 'Megalithic Astronomy: Indications in Standing Stones', in A. Beer (ed.), *Vistas in Astronomy*, VII, 1.

[8] G. S. Hawkins, 'Stonehenge: a Neolithic Computer', *Nature*, 202, 1964, 1258.

[9] P. V. Neugebauer, *Tafeln zur Astronomischen Chronologie*, Leipzig, 1912.