Megalithic geometry in standing stones

Analysis of the work of the Megalithic builders who, 4000 years ago, erected their circles all over Britain, shows that they matched their extraordinary ability in handling massive stones with a knowledge of Pythagorean triangles

by Professor Alexander Thom

IN spite of the comparatively recent wholesale destruction of Megalithic sites there are still the remains of several hundred stone circles in Britain. They are about 4000 years old. Some 140 of them are sufficiently complete for the original plan to be determined with reasonable accuracy. This study involves making accurate large-scale surveys, with the orientations determined astronomically to a tenth of a degree.

From careful statistical analysis of the dimensions of these circles it has been definitely established that the erectors used an accurate unit of length which I propose to call the Megalithic yard. This unit was in use from one end of the country to the other, so that whether it is determined from the English circles or from the Scottish the value turns out to be the same, namely 2.72 feet. My most recent determinations are 2.722 from the English sites and 2.719 from the Scottish, but from the calculated probable errors (about 0.003) the difference is not significant statistically. It is not possible to say that the actual unit used was not a multiple or a submultiple of this: in fact we find half yards used occasionally and with much less certainty quarter yards.

Over a hundred of the rings are true circles varying in diameter from a few yards to 370 feet. The remainder are much more interesting, giving, as they do, an insight into the geometrical knowledge of the erectors. With only one or two exceptions all fall into one or other of the six classes shown in Figure 1.

There are at least 20 "flattened circles" of Type A and 11 of Type B. The geometrical construction used for Type A is fairly obvious and could be set out on level ground with stakes and a rope. A chord equal to the radius subtends an angle of 60° at the centre and so four arcs of this size can be set off from A clockwise to B. The radius at A (and at B) is then divided into two equal parts giving the centres from which the two short arcs are struck. The pattern is then completed by an arc centred on C. Type B is simpler. The diameter is divided into three equal parts and the points so found are used to strike the two short arcs. The pattern is completed, as in Type A, by an arc centred on C. I have published in the *Mathematical Gazette* (Vol. 45, No. 352) and elsewhere accounts of ten surveys of these flattened circles, and anyone interested can see how accurately the constructions were followed.

The ellipse is perhaps the most surprising. The use of the ellipse in Megalithic times has been demonstrated in a recent paper by a Scottish astronomer, Dr A. E. Roy. The best example I have among my surveys is shown in Figure 2. Here the erectors placed two stakes at the foci, C and D, 6 Megalithic yards (MY) apart, and with a loop of rope $22\frac{1}{2}$ MY long, placed exactly over the stakes, proceeded in the usual way to scribe an ellipse on the ground. The periphery of such an ellipse can be calculated and is found to be 50.08 MY. It would hardly be possible on the ground to come any nearer to 50 and it is almost certain that the figure of $22\frac{1}{2}$ for the length of the rope, giving a major axis of $16\frac{1}{2}$, was chosen because it made the periphery what the erectors almost certainly believed was exactly 50. It was no mean achievement to find an ellipse with exactly 6 MY between the foci which had at the same time a major axis that was an integral number of half MYs and a periphery a multiple of 10 MY. The multiple of 10 was evidently important. The rings at Woodhenge for example are set out by a construction which made the peripheries multiples of 20 yards. The circles at The Sanctuary near Avebury have peripheries which are multiples of 5. When we examine the true circles with diameters that are definitely not integral numbers of Megalithic yards we find that in a statistically significant number of cases the peripheries are integral.

Whatever the reason, these people considered it important to make as many of the controlling dimensions as possible multiples of their yard and were prepared to go to a great deal of trouble to achieve this end. This is brought out clearly by a study of the egg-shaped rings of which I have surveys. One of these surveys is reproduced in Figure 3 which shows the northern ring at Clava near Inverness. We see that the construction is based on the 3, 4, 5 triangle, a right-angle

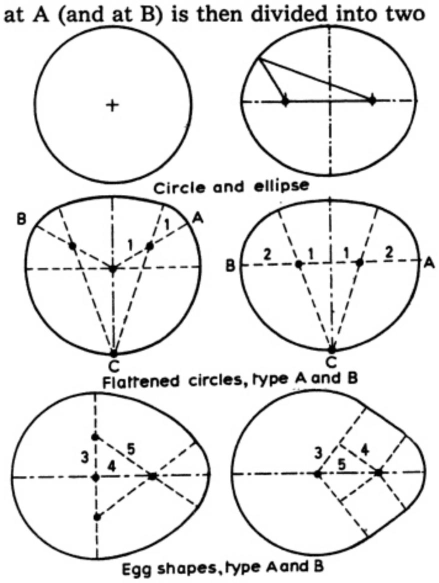

FIGURE 1. *Six main classes of megalithic rings.*

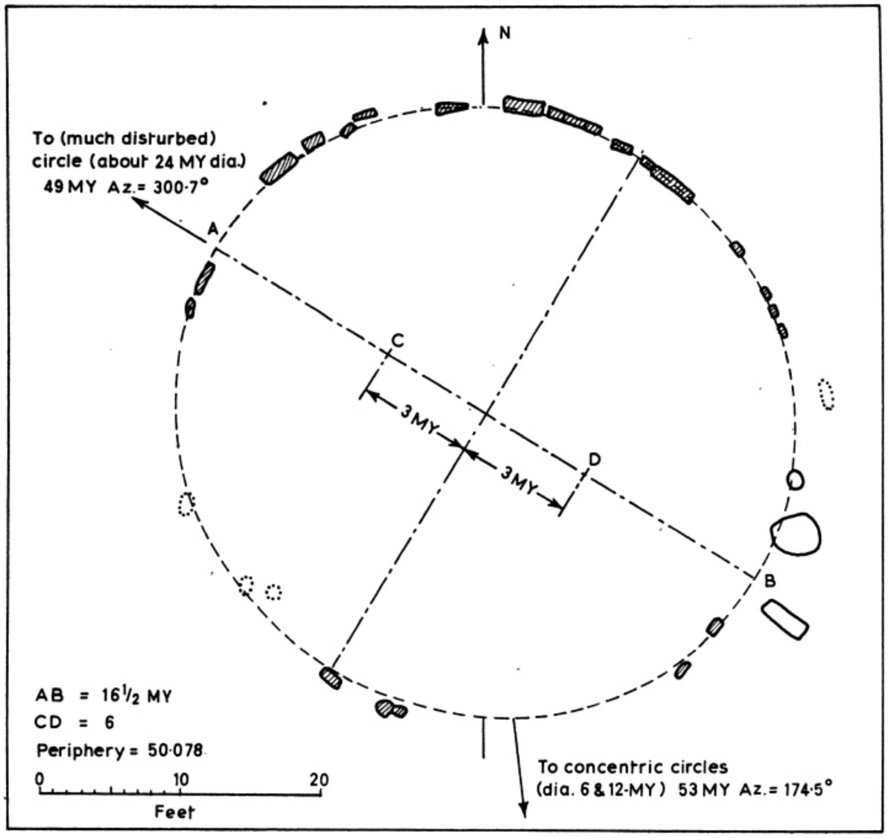

FIGURE 2. *Ellipse at the Sands of Forvie near Newburgh.*

triangle known to the ancients. The corners of two such triangles (in the form 6, 8 and 10 MY) provided the four centres from which the four circular arcs forming the periphery were struck. With the triangles used in this way, if one of the arcs is drawn with an integral value for the radius then it follows that all four radii will be integral. The same idea, but with the triangle turned round, appears in the inner ring of the double circle at Druid Temple which is also near Inverness. The radii here are again exactly whole yards.

The 3, 4, 5 triangle is not the only right-angle triangle which appears in these rings. At the time of writing I have in my surveys egg-shaped rings based on the following values for the sides.

3, 4, 5	$3^2+4^2=5^2$
5, 12, 13	$5^2+12^2=13^2$
12, 35, 37	$12^2+35^2=37^2$
8, 9, 12	$8^2+9^2=12^2+1$
11, 13, 17	$11^2+13^2=17^2+1$
38, 49, 62	$38^2+49^2=62^2+1$

From the published survey of Cairnpaple Hill, the ring there appears to be based on:

28, 30, 41	$28^2+30^2=41^2+3$

It will be seen that three of the above are exact Pythagorean triangles and the others fail by such a small amount that the discrepancy would hardly be apparent on the ground. The greatest discrepancy is in the 8, 9, 12 triangle where the hypotenuse is really 12.04. The others are better approximations.

What conclusions can we draw? These engineers were capable of transporting and erecting stones weighing 20 tons, even in outlying districts like the Hebrides where the population can never have been high. Now we find that they had a knowledge of accurate measurements by a standard yard accurately reproduced and observed throughout England, Wales and Scotland. They had a good working knowledge of elementary geometry and they could measure the length of a curved line with an accuracy better than 0.2 per cent. They certainly knew the properties of the 3, 4, 5 triangle and almost certainly the 5, 12, 13 and 12, 35, 37 triangles. It is perhaps too much to say that they knew Pythagoras's theorem. For them this would have involved a knowledge of arithmetic, and while some students surmise, on totally different grounds, that they had such a knowledge, there is so far no proof that it was sufficient to enable them to square two-figure numbers. Nevertheless we cannot be certain. They wrote their results in stone and it is just possible that these monuments were intended to enshrine an esoteric record of their mathematical achievements.

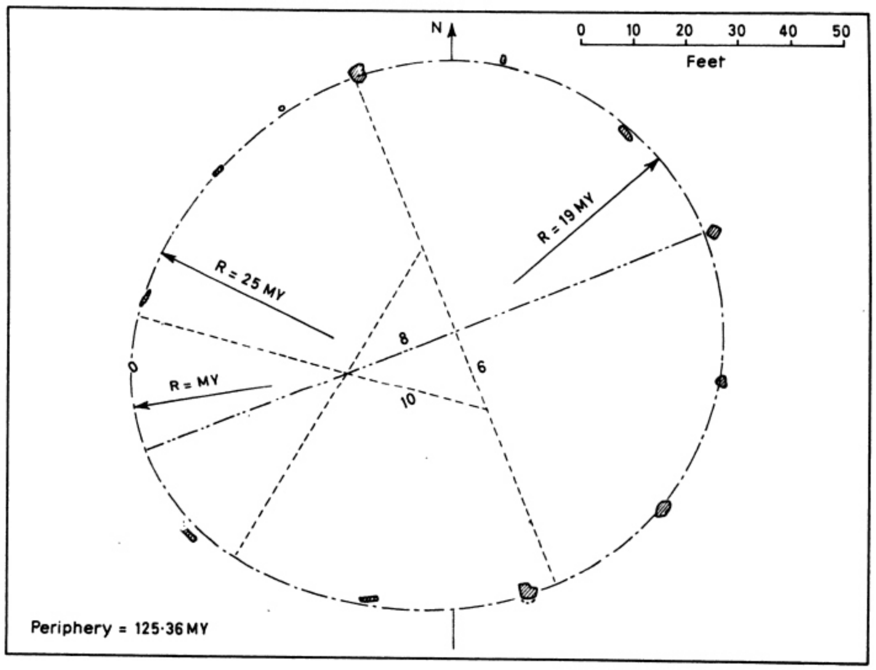

FIGURE 3. *The northern ring at Clava, near Inverness.*