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AVEBURY (1): A NEW ASSESSMENT OF THE GEOMETRY AND METROLOGY OF THE RING

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An authoritative description of the site at Avebury will be found in the volume *Windmill Hill and Avebury* prepared by Dr I. S. Smith to record the excavations and restorations carried out by Alexander Keiller and his associates.[1] Miss Smith deals not only with the ring and the internal constructions but with the bank and ditch and also with the West Kennet Avenue. The main ring itself probably had a perimeter of 3545ft and is estimated to have had originally 98 stones. Some of these are very large, but in 1936 there were only nine upright and another ten lying fallen. The remainder had been destroyed or buried by the villagers. Stukeley has described the methods used for the destruction and indicates that the villagers seemed to take a fiendish delight in breaking up the monument.[2] The destruction would have made it quite impossible to discover the original design were it not for the fact that the stones had been socketed into the underlying chalk. Keiller and his associates were thus able by excavation to find approximately the original positions of some of the stones. His success in pinpointing each exact position depended on the size of the hole in the chalk which in some cases had been enlarged by subsequent disturbances. For example, when the villagers felled a stone they dug out one side of the foundation and toppled the stone across a suitably excavated burning pit in which a fire was lit to destroy the stone. Keiller and his associates have excavated most of the western side of the ring and placed concrete plinths in what they considered to be the original stone positions. But it will be understood from what has been said above that these plinths and indeed also the re-erected stones cannot always have been accurately placed in the original positions and may in some cases have been misplaced by several feet. The eastern side of the ring has not yet been excavated, but a good indication of its position is given by the undisturbed stone no. 68, by two fallen stones and by some burning pits.

In the frontispiece to *Megalithic sites in Britain* a small scale reproduction of a survey of the ring is shown, probably correct to about ± 1 foot.[3] Superimposed on the above plan is an accurate drawing of the arrangement to which we believe the geometry was set out. Essentially this is based on a 3,4,5 triangle in units of 25 Megalithic yards (my) or 10 Megalithic rods.

Professor Kendall in his analysis of the data on circles and rings given in *Megalithic sites in Britain* separated the result obtained for the value of the Megalithic yard in Scotland from that obtained in England.[4] His conclusion was that the Scottish data gave a more convincing value. Because of its complex geometry, Avebury was not included in the English data; by 1967 Stonehenge had not been surveyed by us, nor had Brogar ring been surveyed and analysed for inclusion in the Scottish data.

We consider that because of its size and the fact that we know its geometry, Avebury provides the best opportunity for determining, from a single site in England, the value of the Megalithic yard. Accordingly we decided to make an

Stone	x	y		Δ
1	733·7	44·0		+1·4
3	659·7	28·0		−0·9
4	624·2	19·3	centre *C*	+1·9
5	588·4	13·9	(x′ = 520·9,	+2·4
6	551·6	12·3	y′ = 720·8)	+1·5
7	515·1	9·5		+3·6
8	478·0	16·6		−2·2
9	445·3	23·4		−0·8
10	413·8	46·2		−1·1
11	377·9	74·1		−2·1
12	357·1	94·1		−4·7
13	327·7	112·4		−1·2
14	300·6	136·2		−2·3
15	272·0	158·8	centre *W*	−0·9
16	243·5	183·0	(x′ = 1612·8,	−0·1
17	216·3	205·0	y′ = 1697·1)	+2·2
18	188·9	229·8		+3·1
19	163·5	255·5		+2·7
20	140·0	285·0		−1·1
21	120·6	305·7		−1·2
22	103·1	323·1		−0·2
23	85·9	344·0		−1·3
24	61·8	371·3		−1·1

Stone	x	y		Δ
30	19·3	624·4		+1·4
31	24·9	663·0		+1·6
32	33·3	698·3		+0·4
33	43·7	731·3	centre *A*	−1·4
34	55·5	764·4	(x′ = 723·2,	−2·9
35	62·9	790·1	y′ = 538·6)	−1·1
36	69·2	815·0		+2·3
37	85·0	849·8		+2·3
38	98·5	884·6		+6·4
39	123·6	910·5		−2·1
40	146·8	936·9		−3·5
41	175·2	962·4		−0·3
42	206·7	984·7	centre *B*	+0·6
43	237·6	1002·9	(x′ = 586·6,	+0·3
44	270·3	1022·5	y′ = 386·9)	+2·2
45	292·5	1031·2		+0·5
46	315·8	1042·0		+1·2
50	461·1	1085·4	centre *Z* (x′ = 844·1, y′ = −921·7)	+1·8
68	1033·4	946·2	centre *P* (x′ = 489·2, y′ = 560·6)	−1·3
98	769·9	64·9	centre *D* (x′ = 611·0 y′ = 584·6)	−1·0

TABLE 1. Coordinates (in feet) of stones in Avebury Main Ring. The values given are the estimated centres of each stone referred to an arbitrary zero, x towards east and y towards true geographical north. The table also gives the coordinates (x', y') of the centres of the arcs on which the stones were placed and the displacements $\Delta = \sqrt{((x-x')^2+(y-y')^2)} - R_a$, where R_a is the intended radius, now using 1my = 2·722 ft.

entirely new survey of the site, using methods of greater accuracy, and to make a mathematical analysis of the data obtained.

With steel tapes and a theodolite capable of reading angles to within ± 5 seconds of arc, we ran a closed seven-sided traverse round the ring and checked the azimuth astronomically at three of the stations. The traverse closed to within an inch or two and so we could fix the position of every stone and plinth. It will be understood that to do justice to this new survey it would be necessary to plot it to an unmanageably large scale. Accordingly we calculated from the field survey measurements the coordinates of every stone in the ring and these are given in Table 1. The origin was chosen so as to give positive x and y values over the whole site. The problem then was to find a method whereby a 'least squares' solution would give the value which had most probably been used for the Megalithic yard in setting out the ring. Before we describe how this was done it is necessary to examine the geometry in detail.

The Geometry of the Ring

This is the only ring known to us which consists of circular arcs meeting at an angle instead of running smoothly into one another. It thus has what might

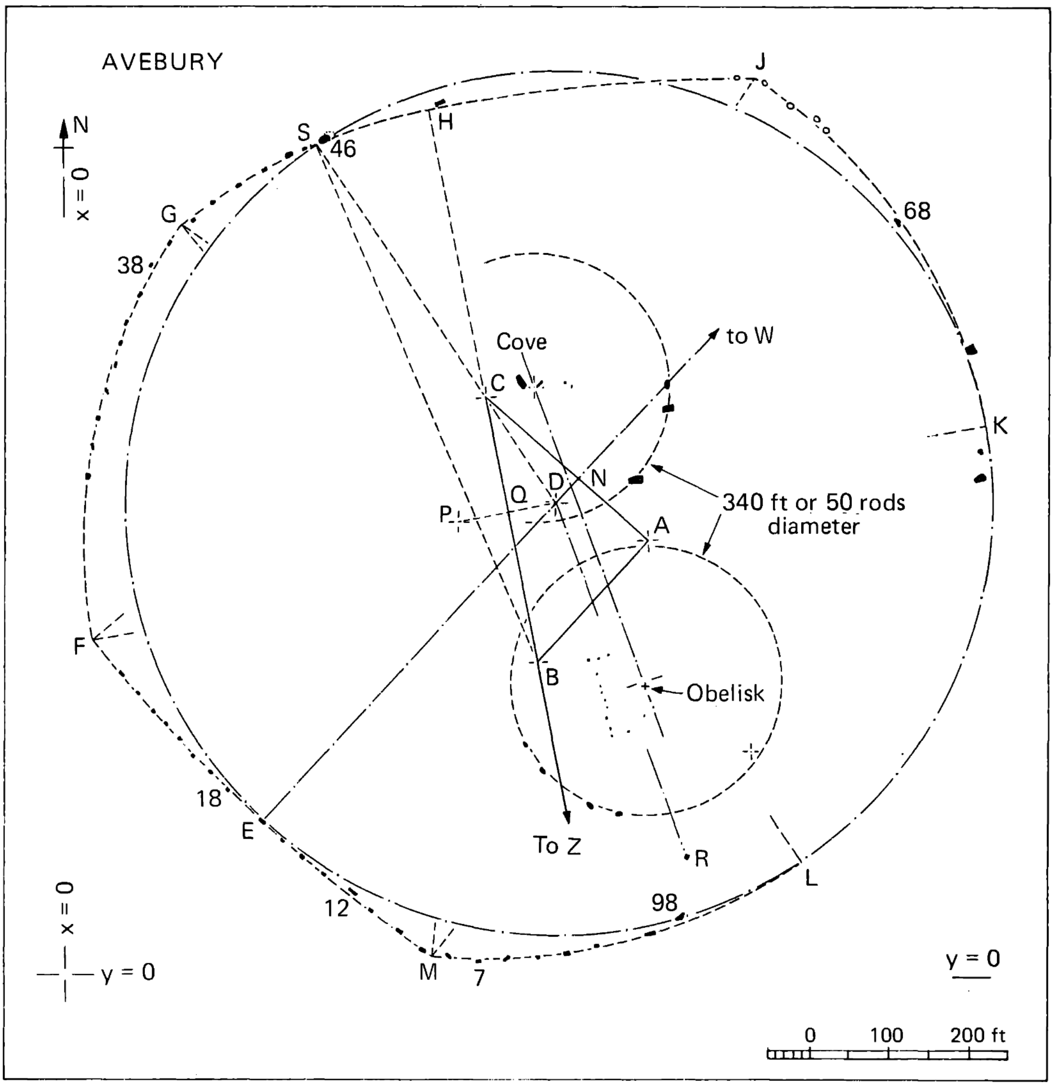

FIG. 1. The Ring of Avebury. Five burning pits are seen adjacent to *J*.

be called corners. An interesting fact is that the distance along the arc between these corners is in every case close to an integral number of rods (see Table 2). It will be understood that to invent a design which has this peculiarity presented a very difficult problem. It is shown later how successful the designers were.

The basic geometry is shown in Figure 1. To set out the ring construct a triangle *ABC* with *AB* 75, *AC* 100 and *CB* 125 my. The angle *A* is obviously a right angle. Now find a point *S*, 260 from *B* and 140 from *C*, produce *SC* to *D* making *CD* 60; *DN* the perpendicular distance to CA should then be 15·061 (presumably assumed to be 15). But the triangle *BSC* is so badly conditioned that it is almost impossible to construct it accurately. Perhaps *DN* was made 15 and *DC* produced to give *S*. *D* is evidently considered as the main centre so, with *D* as centre and radius 200, describe a circle. Now make *PQ* equal to *QD*, the angle at *Q* being a right angle. It seems possible that in an early attempt the designers tried to make the figure symmetrical about *CB* and this

TABLE 2. Lengths of the Avebury arcs, computed from the geometry in Figure 1, assuming (i) $CS = 140$, $CB = 125$ and $SB = 260$my, so that $DN = 15{\cdot}062$; (ii) $CS = 140$, $CD = 60$ and $DN = 15$, so that $SB = 259{\cdot}97$.

Arc	*ME*	*EF*	*FG*	*GH*	*HJ*	*JK*	*KM*	Total
Length (i)	97·16	117·31	199·99	129·68	149·94	195·59	412·63	1302·30
Length (ii)	97·23	117·43	199·87	129·68	149·72	195·75	412·58	1302·26
"Nominal"	97·50	117·50	200·00	130·00	150·00	195·00	412·50	1302·50

may explain the position of *P*. However there is no further indication of symmetry and so we proceed as follows: with centre *A* and radius 260 describe an arc *FG;* similarly with centres *C* and *B* describe arcs *ML* and *GH*, again with radii 260. *EDN* is parallel to *BA* and must be produced until a point *W* is found 750 from *E*. With this centre draw the arc *FM*. Similarly produce *CB* until we find a centre *Z* 750 from *H* and with this describe the arc *HJ*. The remaining arc *JK* is drawn with centre at *P*, *K* being on the line *PQD*. Even an experienced draughtsman will find this design difficult to set out unless the perpendicular distance of *S* from *BC* produced be first calculated.

In view of the importance of the site we have calculated with the greatest care the lengths of the arcs between the corners on two different assumptions: (i) that the triangle *BCS* was set out first, and (ii) that *DN* was made exactly 15. The two assumptions are seen to give practically identical values and every one of the values lies close to an integral multiple of 2½ my, which might be called 'the nominal value' (Table 2). Dr Douglas Heggie has made the (necessarily long) calculations to check the values in Table 2. He has also estimated in three different ways the probability of the near-integral lengths of the arcs and obtained values between 0·1 % and 1·0 %.[5]

Note that the above calculations and the results depend entirely on the assumed geometry and so only indirectly come from the measurement made in the field.

The Results of Field Work

It is now necessary to see how nearly the geometrical construction agrees with the field survey. The geometrical construction was set out carefully on tracing paper assuming 1 my=2·720ft and placed on the survey. It was moved about until the agreement seemed as good as possible. The coordinates of the point *D* were then read off along with the azimuth of *DCS*. From these were calculated the coordinates of all the other centres *A*, *B*, *C*, etc. The next step was to calculate from Table 1 the distance of each stone from the centre of the circular arc on which it stands. Comparing these distances with the intended radius we found the discrepancies (Δ) for each stone (see Table 1). It was proposed from these discrepancies to find the best position for the construction and for the value of the Megalithic yard. There were four unknowns, namely the two corrections to the coordinates of *D*, the correction to the azimuth of *DCS* and the correction to the scale. We had taken 2·720ft to be a Megalithic yard and this might need alteration.

In the *Journal of the Royal Statistical Society* in 1955 the following method[6] for dealing with this problem was suggested:

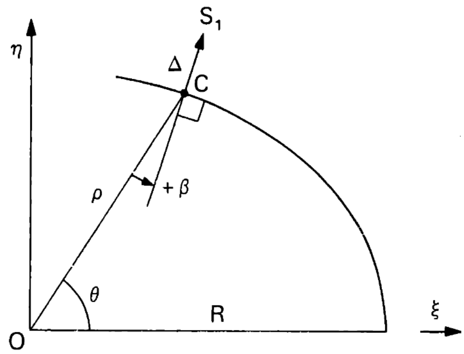

FIG. 2. Definitions of R, ρ, β and Δ.

Figure 2 shows a typical part of the ring. Let the size be specified by R, here taken to be the radius of the main circular part, 200my. $\Delta = CS_1$ is the perpendicular distance of a stone S_1 outside the ring; $\rho = OC$; β=angle between OC and the normal to the ring at C.

Suppose that we scale up the whole size of the outline so that R becomes $R+r$, then OC increases by $\rho r/R$ and Δ decreases by $(\rho r \cos\beta)/R$. Suppose also that the assumed position of O moves a small amount to (ξ, η) and that the whole outline rotates through a small angle ϕ.

Then CS_1 becomes

$$\Delta' = \Delta - r(\rho\cos\beta)/R - \xi\cos(\theta+\beta) - \eta\sin(\theta+\beta) - \phi\rho\sin\beta. \qquad (1)$$

We want S_1 to be on the ring, *i.e.*

$$\Delta' = 0. \qquad (2)$$

Every stone gives us an equation like (2).

There are 4 unknowns to be found; namely the coordinates (ξ, η) of the final position of the centre, the amount (ϕ) the outline has to be rotated and the amount (r) by which its size has to be increased. Thus we have in all 42 equations to solve for 4 unknowns, *i.e.* one equation for each stone and plinth.

We began by assuming (a) the centre D to be at 611·0, 584·0 (feet), (b) the line DCS to be at azimuth 146°42′, (c) the radius of the main ring to be 544ft and (d) the Megalithic yard to be 2·720ft (200×2·720=544). With these values we calculated the coordinates (x', y') of the centres A, B, C, etc. of all the arcs. It was then easy to calculate the distance of each stone from the centre of the arc on which it lies. Subtraction of the nominal radius then gave Δ.

The coefficients in (1) were then found and Equations (2) were solved. As we wanted several solutions the help of Dr Fiona Williams of the University of Glasgow Computing Service was obtained.

The position of stone no. 38 is so far away from the ring that we ignored it, but the first solution (see Table 3) retains all other stones and plinths. Stone 12 seems very far inside the ring and we tried the effect of omitting it; similarly with 7 and 18. We may be completely wrong in assuming 98 to be on the circular arc (with radius 200) produced and so we tried the effect of assigning it to the 260 radius arc. The only stones upright in 1936 were 1, 8, 32, 33, 44, 46, 50, 68 and 98.

On any of the selections tried in Table 3 it will be seen that R needs to be slightly increased. It seems reasonable to take the increase as being about

+0·4ft. Hence R becomes 544·0+0·4ft and since this is 200my, the yard becomes 2·722ft.

Taking the arbitrarily chosen values given in the last line of Table 3 we then calculated the revised positions of the centres A, B, C, etc. (Table 1) and so were able to obtain by calculation the exact position of the arcs. These are shown in Figure 3 plotted in the correct positions. On the large scale of our drawing we could not use trammels to draw the circular arcs and so these were plotted from the equation:

$$y = X^2/2R + X^4/8R^3 + \ldots,$$

where the origin is at the mid point of the arc.

To appreciate fully what is happening here, it is necessary to be quite clear that these arcs are not independent, that is, they have not been independently derived. They are all part of the complete geometrical construction, so that for example if the arc ML is pushed to the north by a foot the arc GH will also be moved by a foot. The computer technique used is analogous to sliding a tracing of the geometry about over a plotted survey to find the best position and orientation. It is more powerful in that it minimizes the sum of the squares of the discrepancies and in that it will as necessary contract or expand the geometrical design. It may be mentioned that the mean Δ in Table 1 is $-0{\cdot}03$ (without Stone 38) and the standard deviation is 1·92. It is unlikely that any other suggested geometrical construction will be found with a much lower standard deviation and so it is safe to assume that we have found the pattern to which the ring was set out.

The accuracy with which the ring has been set out is clearly remarkable; the stones are all close to the arcs. Two of us have considerable experience in setting out roads and railways using modern equipment and we know that, even with the use of such equipment, it would be quite a task to complete this ring satisfactorily. Anyone asked today to do this would begin by making a number of trigonometrical calculations. The erectors did it without theodolites, without steel tapes and without trigonometric tables.

Let us now look at the dimensions which they actually used. Converting to rods we find for the basic triangle 30, 40, 50, BS is 104, CS is 56, CD is 24. The radii of the arcs are 80, 104 and 300. Thus all the dimensions with the exception of PK are integral in rods. Until it is excavated we cannot be certain of the geometry of the east side and so we leave PK at 245·6my.

TABLE 3. Solution of equations with various assumptions.

	ξ (ft)	η (ft)	r (ft)	ϕ (radians)
Omit stone 38	+0·59	+1·15	+0·73	+0·0101
Omit 12 and 38	+0·29	+0·28	+0·43	+0·0019
Omit 7, 12, 18 and 38	−0·23	+0·75	+0·09	+0·0024
Omit 38 and assume that 98 belongs to outer arc	+0·02	+1·25	+0·35	+0·0101
For plotting Figure 3, use arbitrarily	0·00	+0·60	+0·40	+0·0030

Note that the above uses the usual mathematical convention with positive rotation anticlockwise; but azimuth is measured clockwise and so the assumed correction to azimuth is −0·0030 radians or −10 arc minutes.

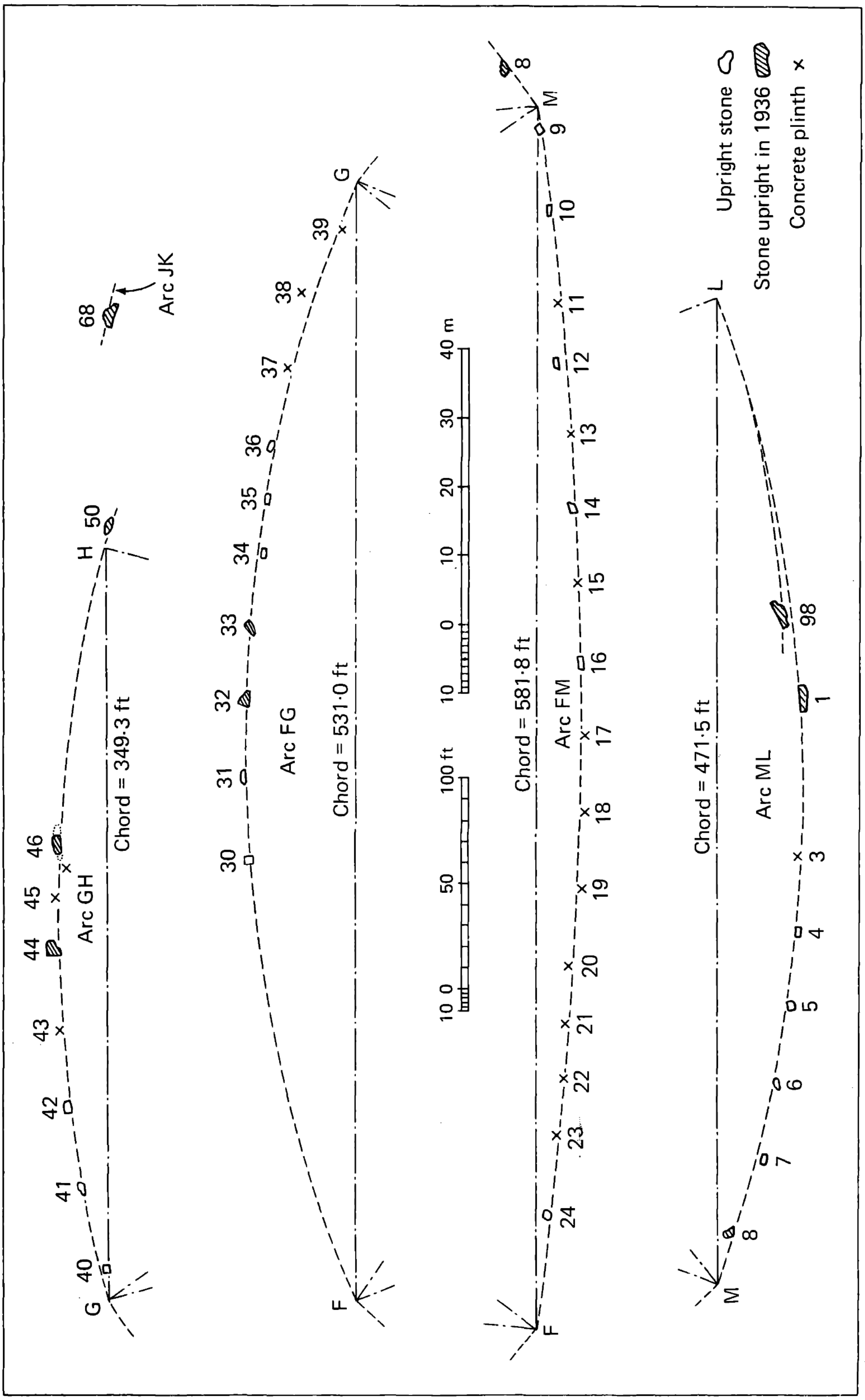
Arc GH
Chord = 349·3 ft
Arc JK
Arc FG
Chord = 531·0 ft
Arc FM
Chord = 581·8 ft
Arc ML
Chord = 471·5 ft
100 ft
40 m
Upright stone
Stone upright in 1936
Concrete plinth

FIG. 3.

We have assumed that Stone 98 lay on the 200 radius (but see Table 3) from D and this is somewhat doubtful because of the direction in which the axis of the stone lies. It is possible that there was another "corner" 105 Megalithic yards along the arc from M. The next arc would then pass nearly through Stone 98, but until this part of the ring has been excavated, this can only be guess work. The assumed arc (Figure 1) runs practically parallel with the edge of the ditch and until one or two stone holes here have been uncovered it seems best to retain the geometry as shown.

The Megalithic Yard

Various claims have been made that the rings were set out by simple pacing but we cannot agree with this hypothesis. The statistical analysis we have made above shows a value of 2·722ft with an error not greater than 0·002 (see Table 3). In our paper on the Brogar Ring we show that the value of the yard used in Orkney was probably 2·725ft.[7] Other papers[8] in this journal show that at Le Menec the yard was 2·721ft and at Kermario 2·724. In view of the great care which was obviously taken to get the Avebury Ring set out to agree exactly with the postulated geometry and in view of the care which we have taken in our survey and analysis we consider that the most accurately determined value of the Megalithic yard in England is that found above, namely 2·722ft or 0·8297m.

How was Avebury set out?

The centres for the flat arcs were outside the ditch and bank. It thus seems *likely* that the ring was laid out before the ditch and bank were constructed. It is difficult to believe that these arcs were set out by means of a long rope. Think of a rope 2040ft long being used in this (by no means level) piece of country! We imagine the rope being supported by a row of men along its length, but even so, what about the stretch in the rope? A varying pull on a 2000ft rope would give considerable differences in length. Such a technique may have been used in the early stages of the design until approximate positions were found, but for the final setting out we consider that the rope can be excluded. Distances were almost certainly set out by two long rods, lifted over one another alternately. After the centre of the arc had been found, the two ends and the mid point of the arc would be set out. This would give the versine or sagitta. One quarter of this sagitta would be the sagitta for the half arcs. Thus five points were found along the arc and as many more subdivisions could be made as were considered necessary. It is quite likely that the arcs with radius 200 and 260 Megalithic yards were set out by the same method. All this presupposes that the design was specified and all the dimensions known. But how was the design discovered? It probably took years of trial and error. It would be an interesting exercise with an electronic computer to try to invent a similar design with all the linear dimensions integral and with the lengths along the arcs between the corners as near to integers as those found in Avebury. Once the difficulty of the problem is fully appreciated one begins to understand how difficult it must have been on the ground.

As a result of our analysis we now know the coordinates of A, B, C and S. The important point S lies just under the overhang of the west end of Stone 46, almost exactly where its position was previously estimated to be. The importance of Stone 46 cannot be exaggerated and yet passing traffic is allowed to knock into it. This has happened at least twice and we have it on good authority that in one case a lorry knocked it well out of the vertical.

It is a disturbing thought that in 1975 we were still allowing the destruction of the Avebury Ring to continue.

Internal Features

We could only record surface features and our own measurements are not sufficient to demonstrate the existence of the northern of the two large inner circles, but ample evidence for the circle is given by Dr Smith.[9] The southern circle is more definite and has a diameter close to 340ft. This is 50 Megalithic rods, and in a previous paper we have shown this to be the exact diameter of the Brogar Ring.[10] It is our belief that the reason for using 50 rods or 125 Megalithic yards is that this diameter makes the circumference 392·7my, very nearly an integral multiple of $2\frac{1}{2}$my. A line joining the centres of the circles is 145my long and if we set off 90my along this line from the south circle and construct a perpendicular 10my long, the end falls on *D*, the main centre of the outer ring.

The ring stone *R* seems to have had some importance. It will be noticed that the line joining the ring stone to the main centre *D* is exactly parallel to the line joining the circle centres, and the azimuth of this line is 340°·2. Standing at the obelisk on the line of the centres and looking through the stones at the Cove, one would be looking along the line to the place where Deneb "set" about 1600 BC. The magnitude of Deneb is 1·3 and we see from Figure 13·1 in *Megalithic sites in Britain* that a star of this magnitude has an extinction angle of about 1°·4. By this we mean that, with even the clearest atmospheric conditions, it is impossible to see the star below an altitude of 1°·4, which is greater than the actual horizon altitude by over 1°. The declination corresponding to azimuth 340°·2, altitude 1°·4, latitude 51°·43 is 36°·8 which is that of Deneb about 1600 BC. The declination of Deneb was however changing due to precession so very slowly that it is not possible to use Deneb for dating. Deneb is one of the first-magnitude stars for which the declination remained constant century after century, and this was probably one of the reasons for the interest obviously taken in this star.[11] The other reason may have been the particularly long run of usefulness of Deneb as a timekeeper throughout the year.[12]

Conclusion

An entirely new and very accurate survey has been made of the ring, and additional detailed calculations have justified the original claim that the geometry of the west side is known exactly. The survey has demonstrated the remarkable accuracy with which the whole ring was set out and has shown the value of the Megalithic yard which was used to be 2·722ft.

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